$$y' = -y + 2xy^2 \Rightarrow y' + y = 2xy^2$$

Bernoulli:

$$y' + P(x)\, y = Q(x)\, y^n$$

$$u = y^{1-n}$$

Tome $u = y^{-1} \Rightarrow u = \dfrac{1}{y}$

$$\Rightarrow y = \frac{1}{u} \Rightarrow y' = -\frac{1}{u^2} \cdot u'$$

e $y^2 = \dfrac{1}{u^2}$

Substituindo: $-\dfrac{1}{u^2} u' + \dfrac{1}{u} = 2x \dfrac{1}{u^2}$

$\times(-u^2)$

$\Rightarrow u' - u = -2x$ (linear com $P(x) = -1$)

Fator integrante: $e^{\int -1\, dx} = e^{-x}$

$$\therefore \underbrace{e^{-x} u' - e^{-x} u}_{} = -2x e^{-x}$$

$$\Rightarrow (e^{-x} \cdot u)' = -2x e^{-x}$$

$$\Rightarrow \int (e^{-x} u)' dx = \int -2x e^{-x} dx \quad \left( \begin{array}{lcl} \text{Por partes:} & & \\ u = -2x & & du = -2dx \\ dv = e^{-x} & \Rightarrow & v = -e^{-x} \end{array} \right)$$

$$\Rightarrow e^{-x} \cdot u + c_1 = 2xe^{-x} - \int 2e^{-x} dx = 2xe^{-x} - 2\int e^{-x} dx$$

$$\Rightarrow e^{-x} u + c_1 = 2x e^{-x} + 2e^{-x} + c_2$$

$$\Rightarrow u = 2x + 2 + ce^{x}$$

Portanto,

$$y = \frac{1}{2x + 2 + ce^{x}}$$

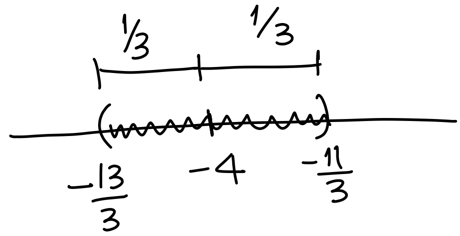
$$\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$$

$$\left|\frac{x_{n+1}}{x_n}\right| = \left| \frac{3^{n+1} \cdot (x+4)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{3^n \cdot (x+4)^n} \right| = \left| \frac{3^{n+1}}{3^n} \cdot \frac{\sqrt{n}}{\sqrt{n+1}} \cdot \frac{(x+4)^{n+1}}{(x+4)^n} \right|$$

$$= \left| 3 \cdot \sqrt{\frac{n}{n+1}} \cdot (x+4) \right| = 3 \sqrt{\frac{n}{n+1}} \cdot |x+4| = 3|x+4| \cdot \sqrt{\frac{n}{n+1} \cdot \frac{\frac{1}{n}}{\frac{1}{n}}}$$

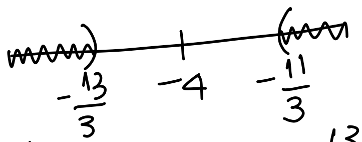
$$= 3|x+4| \sqrt{\frac{1}{1+\frac{1}{n}}} \xrightarrow{n \to \infty} 3|x+4|$$

- $3|x+4| < 1 \Rightarrow |x+4| < \frac{1}{3}$



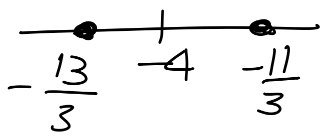

$\therefore$ a série converge se $x \in \left(-\frac{13}{3}, -\frac{11}{3}\right)$

- $3|x+4| > 1 \Rightarrow |x+4| > \frac{1}{3}$




$\therefore$ a série diverge se $x > -\frac{11}{3}$ ou $x < -\frac{13}{3}$

- $3|x+4| = 1 \Rightarrow |x+4| = \frac{1}{3}$




- P/ $x = -\frac{11}{3}$: $\displaystyle\sum_{n=1}^{\infty} \frac{3^n\left(-\frac{11}{3}+4\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n \cdot \left(\frac{1}{3}\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

$= \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/2}}$ (série p, com $p = \frac{1}{2}$) divergente ($p \leq 1$)

- P/ $x = -\frac{13}{3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^n\left(-\frac{13}{3}+4\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n \cdot \left(-\frac{1}{3}\right)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

Teste da série alternada:

- $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

- $\sqrt{n} < \sqrt{n+1}, \ \forall n \in \mathbb{N} \Rightarrow \frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$ (decrescentes)

a série converge.

Portanto, a série original converge para

$x \in \left[-\frac{13}{3}, -\frac{11}{3}\right)$.

$$\frac{n^5 + 2n^3 - 1}{n^4 + 2n^3 + 1} \cdot \frac{\frac{1}{n^5}}{\frac{1}{n^5}}$$

$$= \frac{1 + \frac{2}{n^2} - \frac{1}{n^5}}{\frac{1}{n} + \frac{2}{n^2} + \frac{1}{n^5}} \longrightarrow \infty$$